Lösung: Steckbriefaufgabe

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

WP(0|1) → $f(0)=1$ → I $a \cdot 0^3 + b \cdot 0^2 + c \cdot 0 + d = 1$

$d = 1$

→ $f''(0)=0$ → II $6a \cdot 0 + 2b = 0$

$2b = 0 \quad | :2$

$b = 0$

HP(1|2) → $f(1)=2$ III $a \cdot 1^3 + b \cdot 1^2 + c \cdot 1 + d = 2$

$a + b + c + d = 2 \quad | d=1 \text{ & } b=0$

$a + c + 1 = 2 \quad | -1$

$a + c = 1$

→ $f'(1)=0$ IV $3a \cdot 1^2 + 2b \cdot 1 + c = 0$

$3a + 2b + c = 0 \quad | b=0$

$3a + c = 0$

LGS lösen: III nach a: $a + c = 1 \quad | -c$

$a = 1 - c$

a in IV $3 \cdot (1-c) + c = 0$

$3 - 3c + c = 0 \quad | -3$

$-2c = -3 \quad | \cdot (-2)$

$c = 1{,}5$

$a = 1 - 1{,}5 = -0{,}5$

$f(x) = -0{,}5x^3 + 1{,}5x + 1$

Prüfen der Bed.: $f'(x) = -1{,}5x^2 + 1{,}5$

$f''(x) = -3x \quad \rightarrow f''(1) = -3 \cdot 1 = -3$ ✓ HP(1|2)

$f'''(x) = -3$

$f'''(0) = -3 \neq 0$ ✓ WP(0|1)