Aufgabe:

Berechne das Rotationsvolumen um die y-Achse

a) $f(x) = \frac{1}{4}x$, $[0;1]$

b) $g(x) = 2x - 4$; $[0;2]$

a) $x = \frac{1}{4}y \quad |\cdot 4$

$4x = y = f^{-1}(x)$

$(f^{-1}(x))^2 = (4x)^2 = 16x^2$

$f(0) = \frac{1}{4} \cdot 0 = 0$

$f(1) = \frac{1}{4} \cdot 1 = \frac{1}{4}$

$V = \pi \int_0^{\frac{1}{4}} 16x^2 \, dx$

$= \pi \cdot \left[\frac{16}{3}x^3 \Big|_0^{\frac{1}{4}} \right]$

$= \pi \cdot \left[\frac{16}{3} \cdot \left(\frac{1}{4}\right)^3 - \frac{16}{3} \cdot 0^3 \right]$

$= \pi \cdot \left[\frac{16}{3} \cdot \frac{1}{56} - 0 \right] = \frac{2}{21}\pi$ VE

b) $x = 2y - 4 \quad |+4$

$x + 4 = 2y \quad |:2$

$\frac{1}{2}x + 2 = y = f^{-1}(x)$

$(f^{-1}(x))^2 = \left(\frac{1}{2}x + 2\right)^2 = \frac{1}{4}x^2 + 2x + 4$

$f(0) = 2 \cdot 0 - 4 = 0 - 4 = -4$

$f(2) = 2 \cdot 2 - 4 = 4 - 4 = 0$

$V = \pi \cdot \int_{-4}^{0} \left(\frac{1}{4}x^2 + 2x + 4\right) dx$

$= \pi \left[\frac{1}{12} \cdot x^3 + x^2 + 4x \Big|_{-4}^{0} \right]$

$= \pi \cdot \left[\frac{1}{12} \cdot 0^3 + 0^2 + 4 \cdot 0 - \left(\frac{1}{12} \cdot (-4)^3 + (-4)^2 + 4 \cdot (-4) \right) \right]$

$= \pi \left[0 - \left(-\frac{16}{3} + 16 - 16 \right) \right] = \pi \cdot \left[0 + \frac{16}{3} \right] = \frac{16}{3}\pi$ VE