

Aufgabe:

Berechne die Steigung in x_0 mit der x-Methode:

a) $f(x) = x^2 + x$ in $x_0 = 1$

b) $g(x) = x^3 - 1$ in $x_0 = a$

a) $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$

$$f(1) = 1^2 + 1 = 1 + 1 = 2$$

$$\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1}$$

Polynomdivision: $(x^2 + x - 2) : (x - 1) = x + 2$

$$\begin{array}{r} (x^2 + x - 2) : (x - 1) = x + 2 \\ \underline{-(x^2 - x)} \\ 2x - 2 \\ \underline{-(2x - 2)} \\ 0 \end{array}$$

$$\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1} = \lim_{x \rightarrow 1} \frac{(x-1) \cdot (x+2)}{x-1}$$

$$= \lim_{x \rightarrow 1} x + 2 = 1 + 2 = 3$$

b) $\lim_{x \rightarrow a} \frac{g(x) - g(a)}{x - a}$

$$g(a) = a^3 - 1$$

$$\lim_{x \rightarrow a} \frac{x^3 - 1 - (a^3 - 1)}{x - a}$$

$$\lim_{x \rightarrow a} \frac{x^3 - 1 - a^3 + 1}{x - a}$$

$$\lim_{x \rightarrow a} \frac{x^3 - a^3}{x - a}$$

Polynomdiv: $(x^3 - a^3) : (x - a) = x^2 + ax + a^2$

$$\begin{array}{r} (x^3 - a^3) : (x - a) = x^2 + ax + a^2 \\ \underline{-(x^3 - ax^2)} \\ -a^3 + ax^2 \\ \underline{ax^2 - a^3} \\ -a^3 + a^2x \\ \underline{a^2x - a^3} \\ -a^3 + a^2x \\ \underline{a^2x - a^3} \\ 0 \end{array}$$

$$\lim_{x \rightarrow a} \frac{x^3 - a^3}{x - a} = \lim_{x \rightarrow a} \frac{(x-a) \cdot (x^2 + ax + a^2)}{x - a}$$

$$\lim_{x \rightarrow a} x^2 + ax + a^2 = a^2 + a \cdot a + a^2 = 3a^2$$