Ableitung Wurzelfunktionen:

1) Kettenregel:

a) $f(x) = \sqrt[3]{x^2 - 4x}$

b) $g(x) = \sqrt{x+5}$

c) $h(x) = \sqrt[4]{6x-1} + 3x - 5$

2) Produktregel:

a) $f(x) = x^2 \cdot \sqrt{x}$

b) $g(x) = (x^2 - 1) \cdot \sqrt[3]{x}$

c) $h(x) = (3x^2 - 4x) \cdot \sqrt{x}$

3) Kombination

a) $f(x) = \sqrt{3x+1} \cdot x^2$

b) $g(x) = (4x - 1) \cdot \sqrt{x^2 + 4x}$

c) $h(x) = e^x \cdot \sqrt{2x-1}$

Lösung: Ableitung mit Wurzelfunktionen

Aufgabe 1:

a) $f(x) = \sqrt[3]{x^2 - 4x}$

$u(x) = \sqrt[3]{x} = \sqrt[3]{x^1} = x^{\frac{1}{3}}$ $\quad u'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3} \cdot \frac{1}{x^{\frac{2}{3}}} = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}} = \frac{1}{3 \cdot \sqrt[3]{x^2}}$

$v(x) = x^2 - 4x$ $\quad v'(x) = 2x - 4$

$f'(x) = \frac{1}{3 \cdot \sqrt[3]{(x^2-4x)^2}} \cdot (2x-4) = \frac{2x-4}{3 \cdot \sqrt[3]{(x^2-4x)^2}}$

b) $g(x) = \sqrt{x+5}$

$u(x) = \sqrt{x} = \sqrt[2]{x^1} = x^{\frac{1}{2}}$ $\quad u'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x^1}} = \frac{1}{2\sqrt{x}}$

$v(x) = x + 5$ $\quad v'(x) = 1$

$g'(x) = \frac{1}{2 \cdot \sqrt{x+5}} \cdot 1 = \frac{1}{2\sqrt{x+5}}$

c) $h(x) = \sqrt[4]{6x-1} + 3x - 5$

$u(x) = \sqrt[4]{x} = \sqrt[4]{x^1} = x^{\frac{1}{4}}$ $\quad u'(x) = \frac{1}{4} \cdot x^{-\frac{3}{4}} = \frac{1}{4} \cdot \frac{1}{x^{\frac{3}{4}}} = \frac{1}{4} \cdot \frac{1}{\sqrt[4]{x^3}} = \frac{1}{4 \cdot \sqrt[4]{x^3}}$

$v(x) = 6x - 1$ $\quad v'(x) = 6$

$h'(x) = \frac{1}{4 \cdot \sqrt[4]{(6x-1)^3}} \cdot \frac{6}{1} = \frac{\cancel{6}^{\,2}}{\cancel{4}_{\,3} \cdot \sqrt[4]{(6x-1)^3}} = \frac{2}{3 \cdot \sqrt[4]{(6x-1)^3}}$

Aufgabe 2:

a) $f(x) = x^2 \cdot \sqrt{x}$

$u(x) = x^2$ $\quad$ $u'(x) = 2x$

$v(x) = \sqrt{x} = \sqrt[2]{x^1} = x^{\frac{1}{2}}$ $\quad$ $v'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$

$f'(x) = 2x \cdot \sqrt{x} + x^2 \cdot \frac{1}{2\sqrt{x}}$

b) $g(x) = (x^2 - 1) \cdot \sqrt[3]{x}$

$u(x) = x^2 - 1$ $\quad$ $u'(x) = 2x$

$v(x) = \sqrt[3]{x} = \sqrt[3]{x^1} = x^{\frac{1}{3}}$ $\quad$ $v'(x) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3} \cdot \frac{1}{x^{\frac{2}{3}}} = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}$

$= \frac{1}{3 \cdot \sqrt[3]{x^2}}$

$g'(x) = 2x \cdot \sqrt[3]{x} + (x^2 - 1) \cdot \frac{1}{3 \cdot \sqrt[3]{x^2}}$

c) $h(x) = (3x^2 - 4x) \cdot \sqrt{x}$

$u(x) = 3x^2 - 4x$ $\quad$ $u'(x) = 6x - 4$

$v(x) = \sqrt{x} = \sqrt[2]{x^1} = x^{\frac{1}{2}}$ $\quad$ $v'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$

$h'(x) = (6x - 4) \cdot \sqrt{x} + (3x^2 - 4x) \cdot \frac{1}{2\sqrt{x}}$

Aufgabe 3: $f(x) = \sqrt{3x+1} \cdot x^2$

a) $u(x) = \sqrt{3x+1}$ $\qquad u'(x) = \frac{3}{2\sqrt{3x+1}}$

$v(x) = x^2$ $\qquad v'(x) = 2x$

NR: $\sqrt{3x+1}$

$u(x) = \sqrt{x} = \sqrt[2]{x^1} = x^{\frac{1}{2}}$ $\qquad u'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$

$v(x) = 3x+1$ $\qquad v'(x) = 3$

$\rightarrow \frac{1}{2 \cdot \sqrt{3x+1}} \cdot 3 = \frac{3}{2\sqrt{3x+1}}$

$f'(x) = \frac{3}{2\sqrt{3x+1}} \cdot x^2 + \sqrt{3x+1} \cdot 2x$

$= \frac{3x^2}{2\sqrt{3x+1}} + 2x\sqrt{3x+1}$

b) $g(x) = (4x-1) \cdot \sqrt{x^2+4x}$

$u(x) = 4x-1$ $\qquad u'(x) = 4$

$v(x) = \sqrt{x^2+4x}$ $\qquad v'(x) = \frac{x+2}{\sqrt{x^2+4x}}$

NR $\sqrt{x^2+4x}$

$u(x) = \sqrt{x}$ $\qquad u'(x) = \frac{1}{2\sqrt{x}}$ (siehe oben)

$v(x) = x^2+4x$ $\qquad v'(x) = 2x+4$

$\rightarrow \frac{1}{2 \cdot \sqrt{x^2+4x}} \cdot \frac{(2x+4)}{1} = \frac{2x+4}{2\sqrt{x^2+4x}} = \frac{\cancel{2} \cdot (x+2)}{\cancel{2} \cdot \sqrt{x^2+4x}}$

$g'(x) = 4 \cdot \sqrt{x^2+4x} + (4x-1) \cdot \frac{x+2}{\sqrt{x^2+4x}}$

c) $h(x) = e^x \cdot \sqrt{2x-1}$

$u(x) = e^x$ $\qquad u'(x) = e^x$

$v(x) = \sqrt{2x-1}$ $\qquad v'(x) = \frac{1}{\sqrt{2x-1}}$

NR: $\sqrt{2x-1}$

$u(x) = \sqrt{x}$ $\qquad u'(x) = \frac{1}{2\sqrt{x}}$ $\qquad$ s.o.

$v(x) = 2x-1$ $\qquad v'(x) = 2$

$\rightarrow \quad \frac{1}{2 \cdot \sqrt{2x-1}} \cdot 2 = \frac{\cancel{2}^{1}}{\cancel{2}\sqrt{2x-1}} = \frac{1}{\sqrt{2x-1}}$

$h'(x) = e^x \cdot \sqrt{2x-1} + e^x \cdot \frac{1}{\sqrt{2x-1}}$

$= e^x \left(\sqrt{2x-1} + \frac{1}{\sqrt{2x-1}} \right)$