59. Darstellungswechsel

Natürlich ist es möglich eine Ebene, die in einer bestimmten Form gegeben ist in die beiden anderen Darstellungsformen umzuwandeln:

$$E: \overrightarrow{x} = \overrightarrow{p} + s \cdot \overrightarrow{r_{1}} + t \cdot \overrightarrow{r_{2}}$$
 (Parameter form)

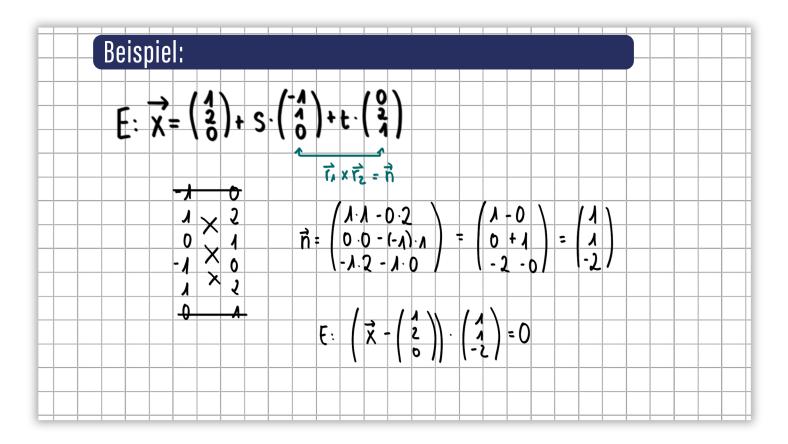
$$E: (\overrightarrow{x} - \overrightarrow{p}) \cdot \overrightarrow{n} = 0$$
 (Normalen form)

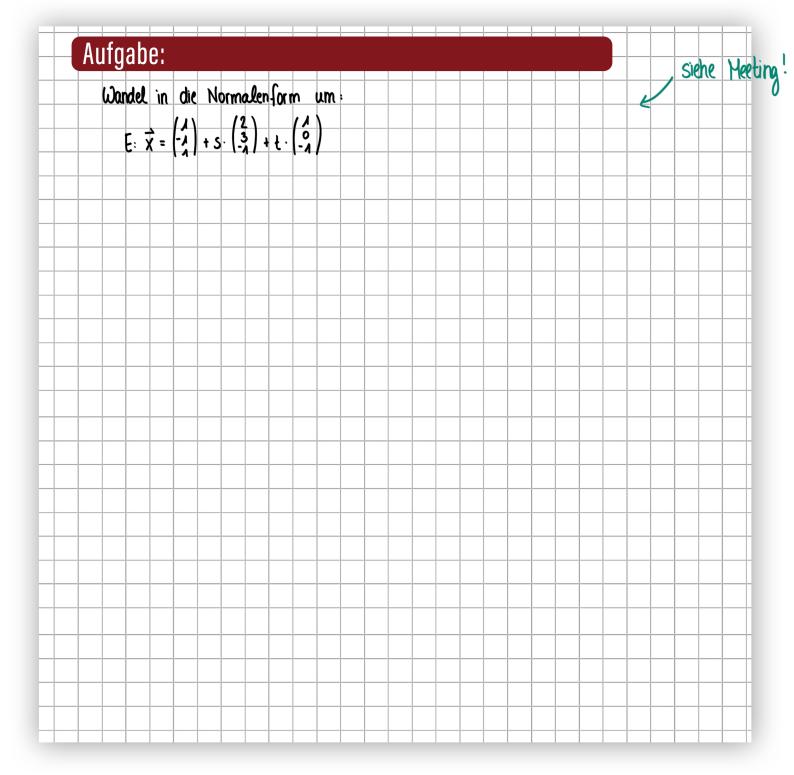
$$E: n_{1}x + n_{2}y + n_{3}z = d$$
 (Koordinaten form)

Umwandlung Parameterform in Normalenform:

$$\begin{bmatrix}
E: \vec{x} = \vec{p} + s \cdot \vec{r_k} + t \cdot \vec{r_k} \\
E: (\vec{x} - \vec{p}) \cdot \vec{n} = 0
\end{bmatrix}$$
der Vektor \vec{p} ist bereits gegeben!

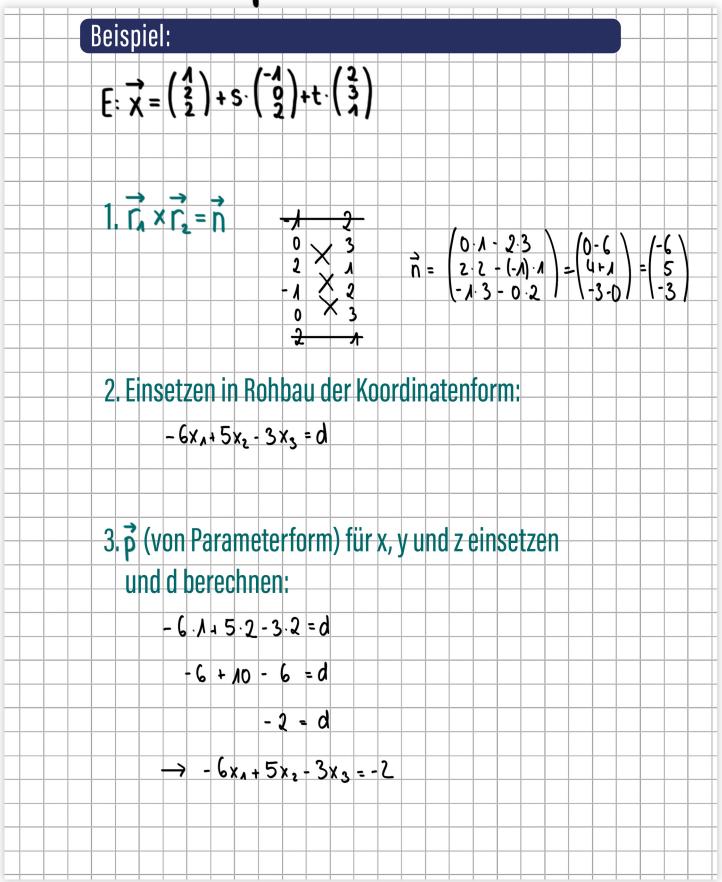
Berechnung des Vektors \vec{n} mithilfe des Kreuzproduktes! $\vec{r}_{1} \times \vec{r}_{2} = \vec{n}$

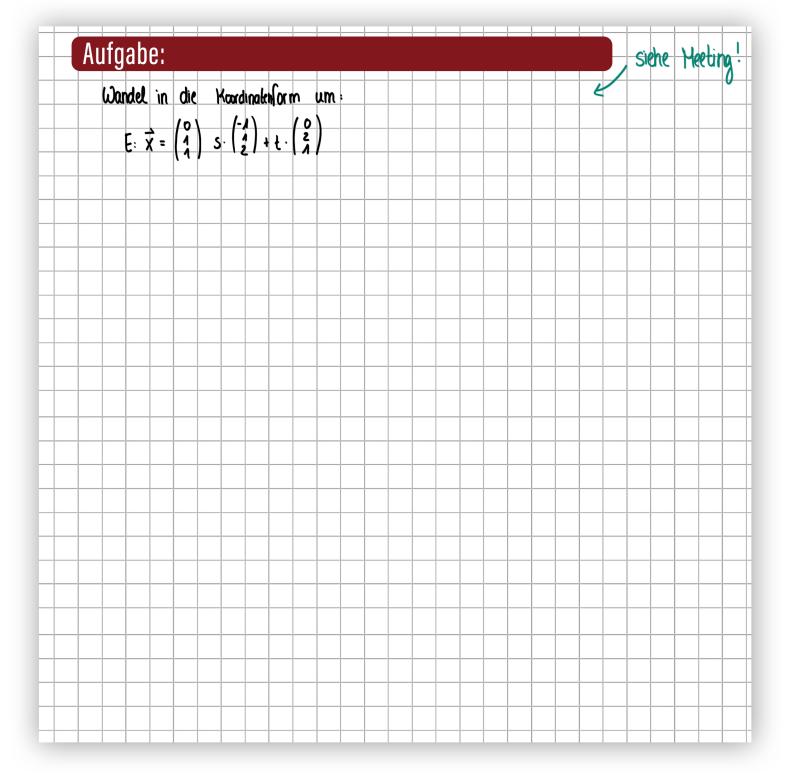




Parameterform in Koordinatenform

$$E: \vec{X} = \vec{p} + s \cdot \vec{r_k} + t \cdot \vec{r_k}$$





Normalenform in Parameterform

$$E: (\vec{x} - \vec{p}) \cdot \vec{n} = 0$$

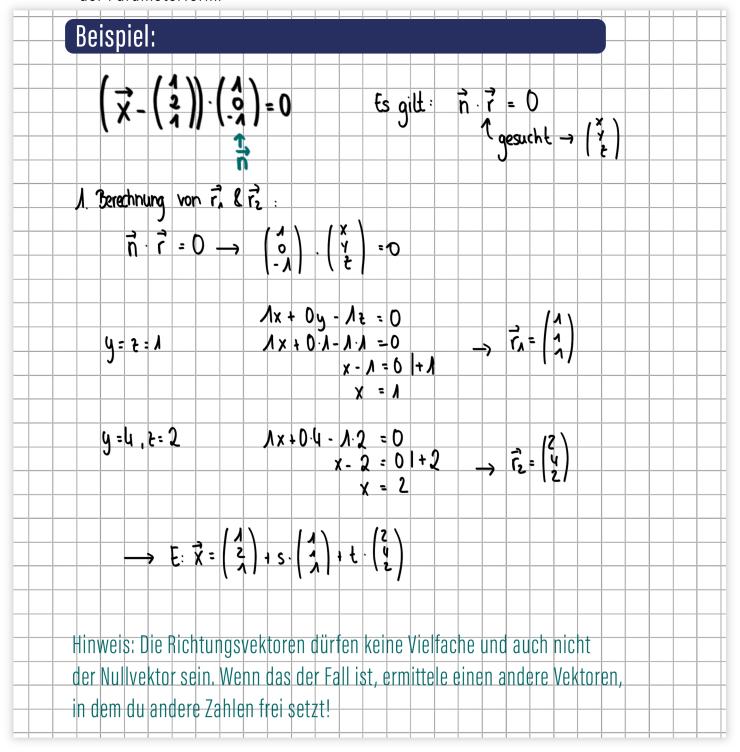
$$E: \vec{x} = \vec{p} + s \cdot \vec{r}_s + t \cdot \vec{r}_s$$

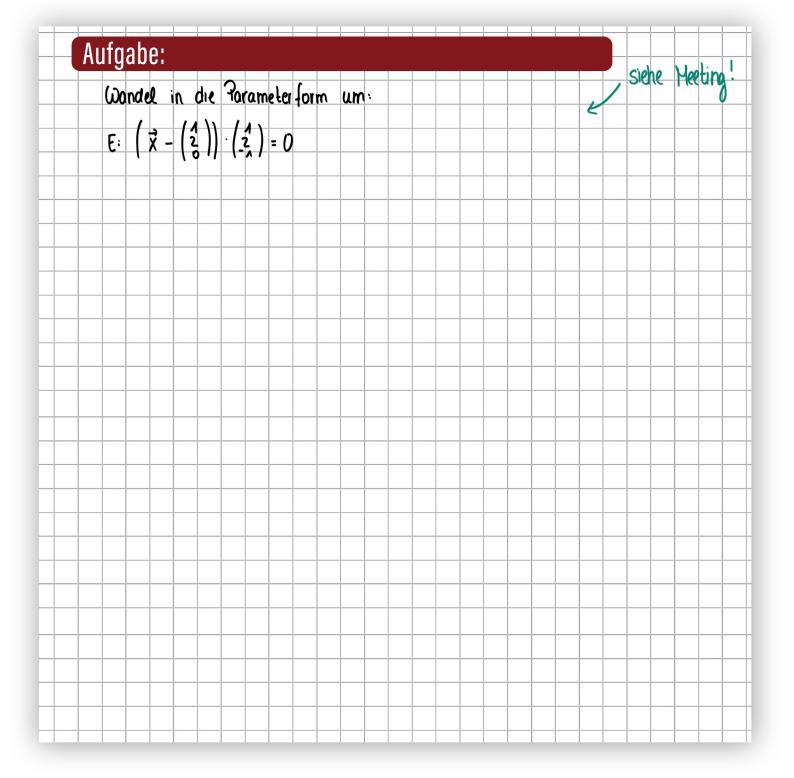
$$I = \vec{r}_s + s \cdot \vec{r}_s + t \cdot \vec{r}_s$$

$$I = \vec{r}_s + s \cdot \vec{r}_s + t \cdot \vec{r}_s$$

$$I = \vec{r}_s + s \cdot \vec{r}_s + t \cdot \vec{r}_s$$

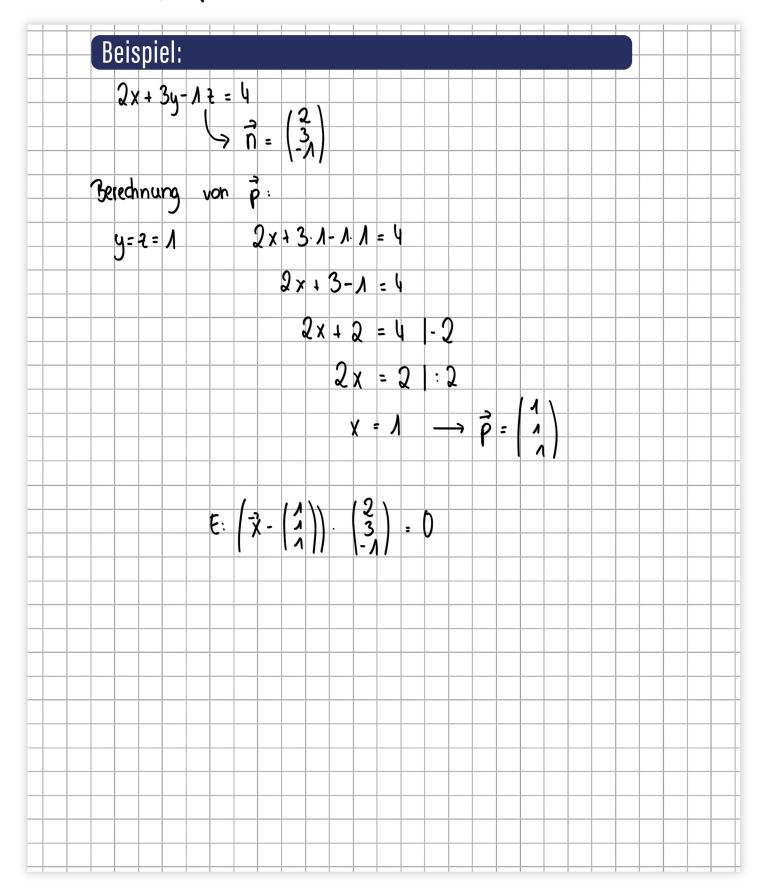
Um die fehlenden Richtungsvektoren zu berechnen, benutzt du den Normalenvektor und suchst zwei Lösungen der Gleichung: $\vec{n} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{0}$ Dabei sind die Lösungen für $\begin{pmatrix} x \\ y \end{pmatrix}$ die gesuchten Richtungsvektoren der Parameterform.

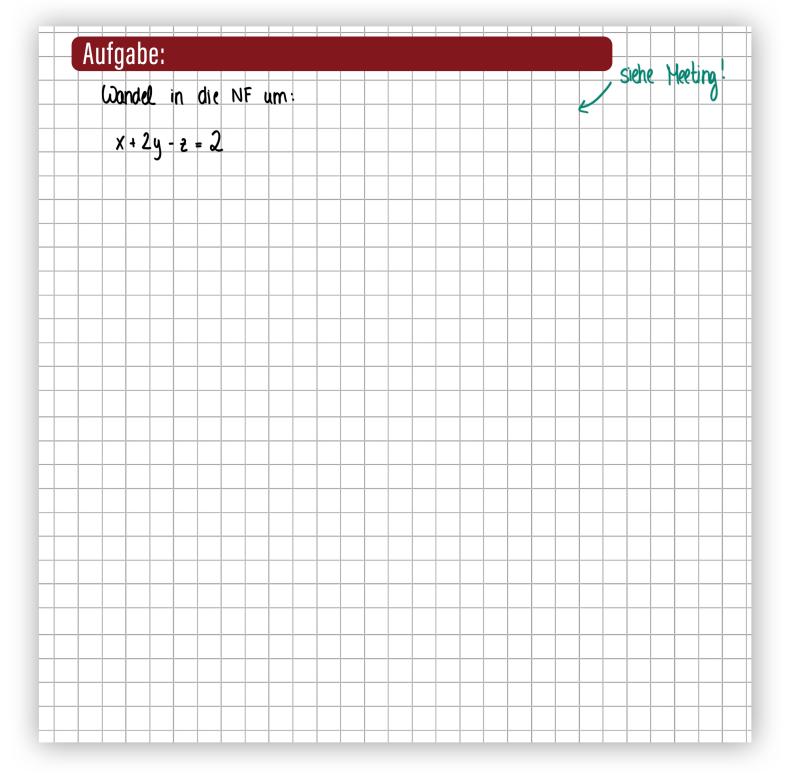




Koordinatenform in Normalenform

E:
$$(\vec{x} - \vec{p}) \cdot \vec{n} = 0$$





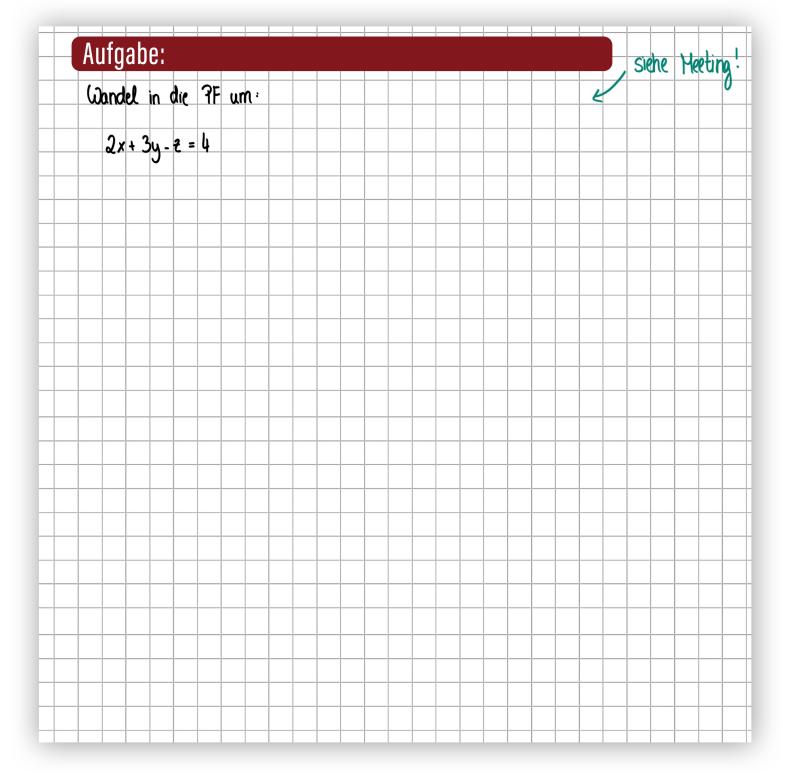
Koordinatenform in Parameterform

E:
$$N_4 \times + N_2 Y_1 + N_3 = d$$

E: $\vec{X} = \vec{p} + s \cdot \vec{r_4} + t \cdot \vec{r_5}$

Hierzu suchst du drei Lösungen der Koordinatenform. Diese lösungen sind Punkte, die auf der Ebene liegen und mithilfe der du anschließend die Parameterform aufstellen kannst!

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Normalenform in Koordinatenform

$$E: (\vec{x} - \vec{b}) \cdot \vec{u} = 0$$

E: N4 x + N24 + N3 Z = d

) ausmultiplizieren

Deinnial			
Beispiel:			
$\left(\overrightarrow{X} - \left(\begin{matrix} A \\ A \end{matrix}\right)\right)$	(-4)-0		
(X-(-X))	(7)-0		
// × \ /	1		
$\left(\left(\begin{array}{c} x \\ y \end{array} \right) - \left(- \left(\begin{array}{c} z \\ z \end{array} \right) \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) - \left(- \left(\begin{array}{c} z \\ z \end{array} \right) \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) = \left(\begin{array}{c} - \left(\begin{array}{c} z \\ z \end{array} \right) = \left(\begin{array}{c} - 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	- 4 + 2	= 0 - 1 - 1	
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	3		



Aufgabe:

Wandel die Parameterform in beide anderen Darstellungsformen um:

$$E: \overrightarrow{X} = \begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$