

Lösungen zum Aufgabenblatt: Unbestimmte Integrale

Hinweis (Regeln):

$$\frac{a}{x^b} = a \cdot x^{-b} \quad \text{und} \quad \sqrt[n]{x^m} = x^{\frac{m}{n}}$$

$$\int c \cdot x^n dx = c \cdot \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1$$

$$f(x) = c \cdot e^{mx+b} \Rightarrow \int f(x) dx = c \cdot \frac{1}{m} e^{mx+b} + C, \quad m \neq 0$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

Aufgabe 1 – Ganzrationale Funktionen

a) $\int (2x^4 - 3x^2 + 5x - 7) dx$

Rechnung:

$$\begin{aligned} \int (2x^4 - 3x^2 + 5x - 7) dx &= \int (2x^4 - 3x^2 + 5x^1 - 7) dx && \text{Exponenten markieren} \\ &= 2 \cdot \frac{1}{4+1} x^{4+1} - 3 \cdot \frac{1}{2+1} x^{2+1} + 5 \cdot \frac{1}{1+1} x^{1+1} - 7x + C && \text{Potenzregel} \\ &= \frac{2}{5} x^5 - x^3 + \frac{5}{2} x^2 - 7x + C \\ &= \boxed{\frac{2}{5} x^5 - x^3 + \frac{5}{2} x^2 - 7x + C} \end{aligned}$$

b) $\int (-x^5 + 4x^3 - 6) dx$

Rechnung:

$$\begin{aligned} \int (-x^5 + 4x^3 - 6) dx &= \int (-x^5 + 4x^3 - 6) dx && \text{Exponenten sichtbar} \\ &= -1 \cdot \frac{1}{5+1} x^{5+1} + 4 \cdot \frac{1}{3+1} x^{3+1} - 6x + C && \text{Potenzregel} \\ &= -\frac{1}{6} x^6 + x^4 - 6x + C \\ &= \boxed{-\frac{1}{6} x^6 + x^4 - 6x + C} \end{aligned}$$

c) $\int (7x^3 - 5x^2 + 2x + 9) dx$

Rechnung:

$$\begin{aligned} \int (7x^3 - 5x^2 + 2x + 9) dx &= \int (7x^3 - 5x^2 + 2x^1 + 9) dx && \text{Exponenten markieren} \\ &= 7 \cdot \frac{1}{3+1} x^{3+1} - 5 \cdot \frac{1}{2+1} x^{2+1} + 2 \cdot \frac{1}{1+1} x^{1+1} + 9x + C && \text{Potenzregel} \\ &= \frac{7}{4} x^4 - \frac{5}{3} x^3 + x^2 + 9x + C \\ &= \boxed{\frac{7}{4} x^4 - \frac{5}{3} x^3 + x^2 + 9x + C} \end{aligned}$$

d) $\int (3x^6 - 8x^2 + x - 4) dx$

Rechnung:

$$\begin{aligned} \int (3x^6 - 8x^2 + x - 4) dx &= \int (3x^6 - 8x^2 + 1x^1 - 4) dx && \text{Exponenten markieren} \\ &= 3 \cdot \frac{1}{6+1} x^{6+1} - 8 \cdot \frac{1}{2+1} x^{2+1} + 1 \cdot \frac{1}{1+1} x^{1+1} - 4x + C && \text{Potenzregel} \\ &= \frac{3}{7} x^7 - \frac{8}{3} x^3 + \frac{1}{2} x^2 - 4x + C \\ &= \boxed{\frac{3}{7} x^7 - \frac{8}{3} x^3 + \frac{1}{2} x^2 - 4x + C} \end{aligned}$$

e) $\int (-4x^4 + 6x^3 - 2x + 1) dx$

Rechnung:

$$\begin{aligned} \int (-4x^4 + 6x^3 - 2x + 1) dx &= \int (-4x^4 + 6x^3 - 2x^1 + 1) dx && \text{Exponenten markieren} \\ &= -4 \cdot \frac{1}{4+1} x^{4+1} + 6 \cdot \frac{1}{3+1} x^{3+1} - 2 \cdot \frac{1}{1+1} x^{1+1} + 1x + C && \text{Potenzregel} \\ &= -\frac{4}{5} x^5 + \frac{3}{2} x^4 - x^2 + x + C \\ &= \boxed{-\frac{4}{5} x^5 + \frac{3}{2} x^4 - x^2 + x + C} \end{aligned}$$

f) $\int (5x^2 - 12x + 3) dx$

Rechnung:

$$\begin{aligned} \int (5x^2 - 12x + 3) dx &= \int (5x^2 - 12x^1 + 3) dx && \text{Exponenten markieren} \\ &= 5 \cdot \frac{1}{2+1} x^{2+1} - 12 \cdot \frac{1}{1+1} x^{1+1} + 3x + C && \text{Potenzregel} \\ &= \frac{5}{3} x^3 - 6x^2 + 3x + C \\ &= \boxed{\frac{5}{3} x^3 - 6x^2 + 3x + C} \end{aligned}$$

g) $\int (9x^5 - 3x^4 + 2x^2 - 1) dx$

Rechnung:

$$\begin{aligned} \int (9x^5 - 3x^4 + 2x^2 - 1) dx &= \int (9x^5 - 3x^4 + 2x^2 - 1) dx && \text{Exponenten sichtbar} \\ &= 9 \cdot \frac{1}{5+1} x^{5+1} - 3 \cdot \frac{1}{4+1} x^{4+1} + 2 \cdot \frac{1}{2+1} x^{2+1} - 1x + C && \text{Potenzregel} \\ &= \frac{3}{2} x^6 - \frac{3}{5} x^5 + \frac{2}{3} x^3 - x + C \\ &= \boxed{\frac{3}{2} x^6 - \frac{3}{5} x^5 + \frac{2}{3} x^3 - x + C} \end{aligned}$$

h) $\int (-2x^3 + x^2 - 10x + 8) dx$

Rechnung:

$$\begin{aligned} \int (-2x^3 + x^2 - 10x + 8) dx &= \int (-2x^3 + x^2 - 10x^1 + 8) dx && \text{Exponenten markieren} \\ &= -2 \cdot \frac{1}{3+1} x^{3+1} + 1 \cdot \frac{1}{2+1} x^{2+1} - 10 \cdot \frac{1}{1+1} x^{1+1} + 8x + C && \text{Potenzregel} \\ &= -\frac{1}{2} x^4 + \frac{1}{3} x^3 - 5x^2 + 8x + C \\ &= \boxed{-\frac{1}{2} x^4 + \frac{1}{3} x^3 - 5x^2 + 8x + C} \end{aligned}$$

i) $\int (4x^7 - x^3 + 6x) dx$

Rechnung:

$$\begin{aligned} \int (4x^7 - x^3 + 6x) dx &= \int (4x^7 - x^3 + 6x^1) dx && \text{Exponenten markieren} \\ &= 4 \cdot \frac{1}{7+1} x^{7+1} - 1 \cdot \frac{1}{3+1} x^{3+1} + 6 \cdot \frac{1}{1+1} x^{1+1} + C && \text{Potenzregel} \\ &= \frac{1}{2} x^8 - \frac{1}{4} x^4 + 3x^2 + C \\ &= \boxed{\frac{1}{2} x^8 - \frac{1}{4} x^4 + 3x^2 + C} \end{aligned}$$

j) $\int (-6x^2 + 11x - 5) dx$

Rechnung:

$$\begin{aligned} \int (-6x^2 + 11x - 5) dx &= \int (-6x^2 + 11x^1 - 5) dx && \text{Exponenten markieren} \\ &= -6 \cdot \frac{1}{2+1} x^{2+1} + 11 \cdot \frac{1}{1+1} x^{1+1} - 5x + C && \text{Potenzregel} \\ &= -2x^3 + \frac{11}{2} x^2 - 5x + C \\ &= \boxed{-2x^3 + \frac{11}{2} x^2 - 5x + C} \end{aligned}$$

Aufgabe 2 – e-Funktion (Basics & lineare Verkettung)

a) $\int 4e^x dx$

Anmerkung: $c = 4, m = 1$

Rechnung:

$$\begin{aligned}\int 4e^x dx &= \int 4e^{1 \cdot x + 0} dx \\ &= 4 \cdot \frac{1}{1} e^{1 \cdot x + 0} + C \\ &= 4e^x + C \\ &= \boxed{4e^x + C}\end{aligned}$$

in Form e^{mx+b}

Regel $f(x) = c \cdot e^{mx+b}$

b) $\int (-7)e^{x-3} dx$

Anmerkung: $c = -7, m = 1$

Rechnung:

$$\begin{aligned}\int (-7)e^{x-3} dx &= \int (-7)e^{1 \cdot x - 3} dx \\ &= (-7) \cdot \frac{1}{1} e^{1 \cdot x - 3} + C \\ &= -7e^{x-3} + C \\ &= \boxed{-7e^{x-3} + C}\end{aligned}$$

in Form e^{mx+b}

Regel $f(x) = c \cdot e^{mx+b}$

c) $\int 6e^{2x+1} dx$

Anmerkung: $c = 6, m = 2$

Rechnung:

$$\begin{aligned}\int 6e^{2x+1} dx &= \int 6e^{2 \cdot x + 1} dx \\ &= 6 \cdot \frac{1}{2} e^{2 \cdot x + 1} + C \\ &= 3e^{2x+1} + C \\ &= \boxed{3e^{2x+1} + C}\end{aligned}$$

in Form e^{mx+b}

Regel $f(x) = c \cdot e^{mx+b}$

d) $\int 3e^{-x+5} dx$

Anmerkung: $c = 3, m = -1$

Rechnung:

$$\begin{aligned}\int 3e^{-x+5} dx &= \int 3e^{(-1) \cdot x + 5} dx \\ &= 3 \cdot \frac{1}{-1} e^{(-1) \cdot x + 5} + C \\ &= -3e^{-x+5} + C \\ &= \boxed{-3e^{-x+5} + C}\end{aligned}$$

in Form e^{mx+b}

Regel $f(x) = c \cdot e^{mx+b}$

e) $\int (-5)e^{-2x-4} dx$

Anmerkung: $c = -5, m = -2$

Rechnung:

$$\begin{aligned}\int (-5)e^{-2x-4} dx &= \int (-5)e^{(-2) \cdot x - 4} dx \\ &= (-5) \cdot \frac{1}{-2} e^{-2x-4} + C \\ &= \frac{5}{2} e^{-2x-4} + C \\ &= \boxed{\frac{5}{2} e^{-2x-4} + C}\end{aligned}$$

in Form e^{mx+b}

Regel $f(x) = c \cdot e^{mx+b}$

f) $\int 8e^{3x-2} dx$

Anmerkung: $c = 8, m = 3$

Rechnung:

$$\begin{aligned}\int 8e^{3x-2} dx &= \int 8e^{3 \cdot x - 2} dx \\ &= 8 \cdot \frac{1}{3} e^{3x-2} + C \\ &= \frac{8}{3} e^{3x-2} + C \\ &= \boxed{\frac{8}{3} e^{3x-2} + C}\end{aligned}$$

in Form e^{mx+b}

Regel $f(x) = c \cdot e^{mx+b}$

g) $\int 2e^{\frac{1}{2}x+6} dx$

Anmerkung: $c = 2, m = \frac{1}{2}$

Rechnung:

$$\begin{aligned}\int 2e^{\frac{1}{2}x+6} dx &= \int 2e^{\frac{1}{2} \cdot x + 6} dx \\ &= 2 \cdot \frac{1}{\frac{1}{2}} e^{\frac{1}{2}x+6} + C \\ &= 4e^{\frac{1}{2}x+6} + C \\ &= \boxed{4e^{\frac{1}{2}x+6} + C}\end{aligned}$$

in Form e^{mx+b}

Regel $f(x) = c \cdot e^{mx+b}$

h) $\int (-9)e^{-\frac{3}{2}x+1} dx$

Anmerkung: $c = -9, m = -\frac{3}{2}$

Rechnung:

$$\begin{aligned}\int (-9)e^{-\frac{3}{2}x+1} dx &= \int (-9)e^{(-\frac{3}{2}) \cdot x+1} dx \\ &= (-9) \cdot \frac{1}{-\frac{3}{2}} e^{-\frac{3}{2}x+1} + C \\ &= 6e^{-\frac{3}{2}x+1} + C \\ &= \boxed{6e^{-\frac{3}{2}x+1} + C}\end{aligned}$$

in Form e^{mx+b}

Regel $f(x) = c \cdot e^{mx+b}$

i) $\int 5e^{\frac{1}{3}x-2} dx$

Anmerkung: $c = 5, m = \frac{1}{3}$

Rechnung:

$$\begin{aligned}\int 5e^{\frac{1}{3}x-2} dx &= \int 5e^{\frac{1}{3} \cdot x-2} dx \\ &= 5 \cdot \frac{1}{\frac{1}{3}} e^{\frac{1}{3}x-2} + C \\ &= 15e^{\frac{1}{3}x-2} + C \\ &= \boxed{15e^{\frac{1}{3}x-2} + C}\end{aligned}$$

in Form e^{mx+b}

Regel $f(x) = c \cdot e^{mx+b}$

j) $\int (-2)e^{4x} dx$

Anmerkung: $c = -2, m = 4$

Rechnung:

$$\begin{aligned}\int (-2)e^{4x} dx &= \int (-2)e^{4 \cdot x+0} dx \\ &= (-2) \cdot \frac{1}{4} e^{4x} + C \\ &= -\frac{1}{2} e^{4x} + C \\ &= \boxed{-\frac{1}{2} e^{4x} + C}\end{aligned}$$

in Form e^{mx+b}

Regel $f(x) = c \cdot e^{mx+b}$

Aufgabe 3 – Wurzeln & einfache Brüche

a) $\int \sqrt{x} \, dx$

Rechnung:

$$\begin{aligned}\int \sqrt{x} \, dx &= \int x^{\frac{1}{2}} \, dx && \text{als Potenz schreiben} \\ &= \frac{1}{\frac{1}{2} + 1} x^{\frac{1}{2} + 1} + C && \text{Potenzregel} \\ &= \frac{1}{\frac{3}{2}} x^{\frac{3}{2}} + C \\ &= \frac{2}{3} x^{\frac{3}{2}} + C \\ &= \boxed{\frac{2}{3} x^{\frac{3}{2}} + C}\end{aligned}$$

b) $\int \sqrt{x^3} \, dx$

Rechnung:

$$\begin{aligned}\int \sqrt{x^3} \, dx &= \int x^{\frac{3}{2}} \, dx && \text{als Potenz schreiben} \\ &= \frac{1}{\frac{3}{2} + 1} x^{\frac{3}{2} + 1} + C && \text{Potenzregel} \\ &= \frac{1}{\frac{5}{2}} x^{\frac{5}{2}} + C \\ &= \frac{2}{5} x^{\frac{5}{2}} + C \\ &= \boxed{\frac{2}{5} x^{\frac{5}{2}} + C}\end{aligned}$$

c) $\int \frac{1}{\sqrt{x}} \, dx$

Rechnung:

$$\begin{aligned}\int \frac{1}{\sqrt{x}} \, dx &= \int x^{-\frac{1}{2}} \, dx && \text{als Potenz schreiben} \\ &= \frac{1}{-\frac{1}{2} + 1} x^{-\frac{1}{2} + 1} + C && \text{Potenzregel} \\ &= \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} + C \\ &= 2x^{\frac{1}{2}} + C \\ &= \boxed{2\sqrt{x} + C}\end{aligned}$$

d) $\int \sqrt[3]{x^2} dx$

Rechnung:

$$\begin{aligned}\int \sqrt[3]{x^2} dx &= \int x^{\frac{2}{3}} dx \\ &= \frac{1}{\frac{2}{3} + 1} x^{\frac{2}{3} + 1} + C \\ &= \frac{1}{\frac{5}{3}} x^{\frac{5}{3}} + C \\ &= \frac{3}{5} x^{\frac{5}{3}} + C \\ &= \boxed{\frac{3}{5} x^{\frac{5}{3}} + C}\end{aligned}$$

als Potenz schreiben

Potenzregel

e) $\int \frac{5}{x^2} dx$

Rechnung:

$$\begin{aligned}\int \frac{5}{x^2} dx &= \int 5x^{-2} dx \\ &= 5 \cdot \frac{1}{-2 + 1} x^{-2+1} + C \\ &= 5 \cdot \frac{1}{-1} x^{-1} + C \\ &= -5x^{-1} + C \\ &= \boxed{-\frac{5}{x} + C}\end{aligned}$$

als Potenz schreiben

Potenzregel

f) $\int \frac{2}{x^3} dx$

Rechnung:

$$\begin{aligned}\int \frac{2}{x^3} dx &= \int 2x^{-3} dx \\ &= 2 \cdot \frac{1}{-3 + 1} x^{-3+1} + C \\ &= 2 \cdot \frac{1}{-2} x^{-2} + C \\ &= -x^{-2} + C \\ &= \boxed{-\frac{1}{x^2} + C}\end{aligned}$$

als Potenz schreiben

Potenzregel

g) $\int \frac{9}{x^4} dx$

Rechnung:

$$\begin{aligned}\int \frac{9}{x^4} dx &= \int 9x^{-4} dx \\ &= 9 \cdot \frac{1}{-4+1} x^{-4+1} + C \\ &= 9 \cdot \frac{1}{-3} x^{-3} + C \\ &= -3x^{-3} + C \\ &= \boxed{-\frac{3}{x^3} + C}\end{aligned}$$

als Potenz schreiben

Potenzregel

h) $\int 7x^{-\frac{1}{2}} dx$

Rechnung:

$$\begin{aligned}\int 7x^{-\frac{1}{2}} dx &= 7 \cdot \frac{1}{-\frac{1}{2}+1} x^{-\frac{1}{2}+1} + C \\ &= 7 \cdot \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} + C \\ &= 14x^{\frac{1}{2}} + C \\ &= \boxed{14\sqrt{x} + C}\end{aligned}$$

Potenzregel

i) $\int -3x^{-\frac{5}{2}} dx$

Rechnung:

$$\begin{aligned}\int -3x^{-\frac{5}{2}} dx &= -3 \cdot \frac{1}{-\frac{5}{2}+1} x^{-\frac{5}{2}+1} + C \\ &= -3 \cdot \frac{1}{-\frac{3}{2}} x^{-\frac{3}{2}} + C \\ &= 2x^{-\frac{3}{2}} + C \\ &= \boxed{\frac{2}{x^{\frac{3}{2}}} + C}\end{aligned}$$

Potenzregel

j) $\int \frac{1}{x} dx$

Rechnung:

$$\begin{aligned}\int \frac{1}{x} dx &= \int x^{-1} dx \\ &= \ln |x| + C \\ &= \boxed{\ln |x| + C}\end{aligned}$$

hier gilt Sonderregel

Regel $\int \frac{1}{x} dx = \ln |x| + C$

Aufgabe 4 – Mix

a) $\int (3x^2 + 2e^x) dx$

Rechnung:

$$\begin{aligned}\int (3x^2 + 2e^x) dx &= \int (3x^2) dx + \int 2e^x dx \\ &= 3 \cdot \frac{1}{2+1} x^{2+1} + 2e^x + C \\ &= x^3 + 2e^x + C \\ &= \boxed{x^3 + 2e^x + C}\end{aligned}$$

b) $\int (5e^{-x} + 4x - 1) dx$

Rechnung:

$$\begin{aligned}\int (5e^{-x} + 4x - 1) dx &= \int 5e^{-x} dx + \int (4x - 1) dx \\ &= 5 \cdot \frac{1}{-1} e^{-x} + 4 \cdot \frac{1}{1+1} x^{1+1} - x + C \\ &= -5e^{-x} + 2x^2 - x + C \\ &= \boxed{-5e^{-x} + 2x^2 - x + C}\end{aligned}$$

c) $\int \left(\frac{6}{x^2} - 2x^3 \right) dx$

Rechnung:

$$\begin{aligned}\int \left(\frac{6}{x^2} - 2x^3 \right) dx &= \int (6x^{-2} - 2x^3) dx && \text{Bruch als Potenz} \\ &= 6 \cdot \frac{1}{-2+1} x^{-2+1} - 2 \cdot \frac{1}{3+1} x^{3+1} + C \\ &= -6x^{-1} - \frac{1}{2} x^4 + C \\ &= \boxed{-\frac{6}{x} - \frac{1}{2} x^4 + C}\end{aligned}$$

d) $\int (\sqrt{x} + 3x) dx$

Rechnung:

$$\begin{aligned}\int (\sqrt{x} + 3x) dx &= \int (x^{\frac{1}{2}} + 3x^1) dx && \text{als Potenzen schreiben} \\ &= \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + 3 \cdot \frac{1}{1+1} x^{1+1} + C \\ &= \frac{2}{3} x^{\frac{3}{2}} + \frac{3}{2} x^2 + C \\ &= \boxed{\frac{2}{3} x^{\frac{3}{2}} + \frac{3}{2} x^2 + C}\end{aligned}$$

e) $\int (2e^{2x-1} - 7) dx$

Rechnung:

$$\begin{aligned}\int (2e^{2x-1} - 7) dx &= \int 2e^{2x-1} dx - \int 7 dx \\ &= 2 \cdot \frac{1}{2} e^{2x-1} - 7x + C \\ &= e^{2x-1} - 7x + C \\ &= \boxed{e^{2x-1} - 7x + C}\end{aligned}$$

f) $\int \left(4x^4 - \frac{3}{x^3}\right) dx$

Rechnung:

$$\begin{aligned}\int \left(4x^4 - \frac{3}{x^3}\right) dx &= \int (4x^4 - 3x^{-3}) dx \\ &= 4 \cdot \frac{1}{4+1} x^{4+1} - 3 \cdot \frac{1}{-3+1} x^{-3+1} + C \\ &= \frac{4}{5} x^5 + \frac{3}{2} x^{-2} + C \\ &= \boxed{\frac{4}{5} x^5 + \frac{3}{2x^2} + C}\end{aligned}$$

Bruch als Potenz

g) $\int (-6e^{-\frac{1}{2}x+2} + x^2) dx$

Rechnung:

$$\begin{aligned}\int (-6e^{-\frac{1}{2}x+2} + x^2) dx &= \int (-6e^{-\frac{1}{2}x+2}) dx + \int x^2 dx \\ &= (-6) \cdot \frac{1}{-\frac{1}{2}} e^{-\frac{1}{2}x+2} + \frac{1}{2+1} x^{2+1} + C \\ &= 12e^{-\frac{1}{2}x+2} + \frac{1}{3} x^3 + C \\ &= \boxed{\frac{1}{3} x^3 + 12e^{-\frac{1}{2}x+2} + C}\end{aligned}$$

h) $\int \left(\frac{8}{x} + 2x^5\right) dx$

Rechnung:

$$\begin{aligned}\int \left(\frac{8}{x} + 2x^5\right) dx &= \int \frac{8}{x} dx + \int 2x^5 dx \\ &= 8 \ln |x| + 2 \cdot \frac{1}{5+1} x^{5+1} + C \\ &= 8 \ln |x| + \frac{1}{3} x^6 + C \\ &= \boxed{\frac{1}{3} x^6 + 8 \ln |x| + C}\end{aligned}$$

i) $\int (3\sqrt[3]{x} - 5x^2 + 1) dx$

Rechnung:

$$\begin{aligned}
 \int (3\sqrt[3]{x} - 5x^2 + 1) dx &= \int (3x^{\frac{1}{3}} - 5x^2 + 1) dx && \text{als Potenzen schreiben} \\
 &= 3 \cdot \frac{1}{\frac{1}{3} + 1} x^{\frac{1}{3} + 1} - 5 \cdot \frac{1}{2 + 1} x^{2 + 1} + 1x + C \\
 &= 3 \cdot \frac{1}{\frac{4}{3}} x^{\frac{4}{3}} - \frac{5}{3} x^3 + x + C \\
 &= \frac{9}{4} x^{\frac{4}{3}} - \frac{5}{3} x^3 + x + C \\
 &= \boxed{\frac{9}{4} x^{\frac{4}{3}} - \frac{5}{3} x^3 + x + C}
 \end{aligned}$$

j) $\int \left(e^{3x+4} - \frac{2}{x^2} \right) dx$

Rechnung:

$$\begin{aligned}
 \int \left(e^{3x+4} - \frac{2}{x^2} \right) dx &= \int e^{3x+4} dx - \int 2x^{-2} dx && \text{Bruch als Potenz} \\
 &= 1 \cdot \frac{1}{3} e^{3x+4} - 2 \cdot \frac{1}{-2 + 1} x^{-2+1} + C \\
 &= \frac{1}{3} e^{3x+4} + 2x^{-1} + C \\
 &= \boxed{\frac{1}{3} e^{3x+4} + \frac{2}{x} + C}
 \end{aligned}$$